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PAPER

Constitutive modeling for the flow stress behaviors of alloys based on variable order fractional derivatives

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E-mail: rfmeng@gzhu.edu.cn**Keywords:** variable order fractional calculus, alloys, flow stress behavior, strain softening, strain hardening

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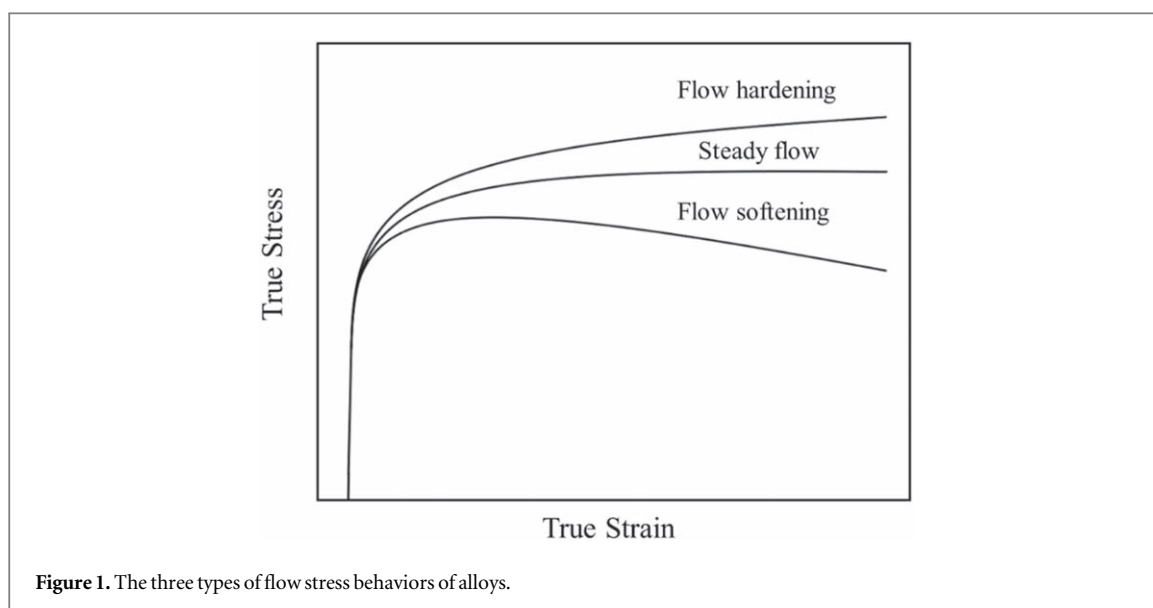
**Abstract**

During hot working, alloys may experience three kinds of flow stress behaviors, including strain hardening, strain softening, or steady flow, because of the competition of work hardening and thermal softening. Modelling the flow stress behaviors plays an essential role in understanding the mechanical properties of alloys. In this paper, the variable order fractional model is provided to describe the flow stress behaviors of alloys. The variation of the fractional order between 0 and 1 can reflect the mechanical property changing between solids and fluids. By assuming that the fractional order varies linearly with time, the proposed model can describe both the strain softening and strain hardening behaviors of alloys. The model fitting results are compared to the experimental data of A356 alloy for strain softening and Cu-Cr-Mg alloy for strain hardening under different temperatures and strain rates. It is validated that the variable order fractional model can accurately describe the flow stress behaviors of alloys. Furthermore, the rule of the variable order is also discussed to analyze its overall values and the changes before and after the yield point. It is concluded that the variation of the fractional order can intuitively reveal the changes in mechanical properties in the flow stress behaviors of alloys, including both strain softening and strain hardening.

1. Introduction

In the forming process, hot working has an important influence on the mechanical properties of alloys. Nowadays, finite element methods have been commonly used to design hot working processes and optimize deformation parameters [1]. The reliability of simulation results significantly depends on the accuracy of the constitutive equation for the flow stresses of alloys under hot deformation. The flow stress behaviors of alloys during hot working are usually complicated due to the combined effects of strain, strain rate, and the forming temperature [2]. Generally, there are three kinds of flow stress curves, as shown in figure 1. The flow stress may exhibit flow hardening, flow softening or steady flow considering the competition of work hardening and thermal softening [3]. After the yield point, if the effect of work hardening plays a dominant role, the flow stress will gradually increase, which is the flow hardening behavior. If the predominant deformation mechanism is thermal softening, the flow softening will appear with decreasing flow stress after a peak stress value. When work hardening and thermal softening reach equilibrium, the stress curve will show a steady flow. Therefore, constitutive modelling for the flow stress behaviors of alloys should have the ability to describe these conditions.

Up to now, a lot of constitutive models have been raised for describing the flow behaviors of alloys. Based on the newest review studies [4, 5], the existing models of flow stress can be divided into three main categories, phenomenological, physical-based, and artificial neural network (ANN) models. Among them, the phenomenological models are considered more efficient, using mathematical functions to characterize the flow stress based on empirical observations. For example, the most widely known Johnson-Cook (JC) model is the polynomial function of strain, strain rate, and forming temperature [6]. Although it benefits from few parameters and low fitted complexity, the coupling effects of strain, strain rate, and temperature are not considered in the JC model. To reflect the coupling effects, the Arrhenius equation is a practical method that



expresses the flow stress by hyperbolic law [7]. However, the accuracy of phenomenological models is currently difficult to meet the simulation requirements of advanced manufacturing processes [8].

On the other hand, the physical-based models account for the physical aspects of alloys, such as dislocation dynamics, thermal activation, and so on [9]. Compared to phenomenological models, they usually require larger amounts of material constants, which limits their applications in simulation. Lastly, the ANN model is an entirely different and new approach able to describe the flow stress behaviors effectively and accurately [10]. But it does not provide mathematical formulas or physical insights into alloys. Therefore, there is still a lack of constitutive models to calculate the flow stress behaviors of alloys practically and accurately, especially to describe both strain softening and strain hardening in the same form.

In recent decades, fractional order calculus has gradually been recognized as a mathematical tool that can effectively describe complex physical phenomena [11]. Fractional order calculus is particularly suitable for describing the intermediate mechanical properties of viscoelastic materials and can significantly reduce the number of model parameters [12]. Considering the viscoelastic properties of alloys, the applications of fractional models have already been studied. For example, Mao *et al* [13] used the modified fractional Maxwell model for the M2052 alloy under the condition of uniaxial tensile test at constant strain rates. Fan and Huang [14] proposed the fractional Burgers model to depict the creep and creep-recovery behavior of 332 aluminum alloy. Around the beginning of the 21st century, a further generalization of the variable order fractional calculus theory was introduced independently by several researchers [15–17]. The variable order fractional operators allow the order to be a function of an independent variable, such as space or time, which provides a novel approach to reflect the evolution of mechanical properties during the deformation process. Ramirez and Coimbra [18] explained the physical significance of the variable fractional order as the rate of disorder inside the material and developed a variable order fractional constitutive model for viscoelastic materials.

The variable order fractional model has shown great potential for applications in many time-varying physical processes [19, 20]. In our previous studies, we have successfully used the variable order fractional model to describe the strain softening [21] and strain hardening [22] behaviors. However, in these works, the strain softening and strain hardening behaviors are studied separately for different materials. There are still few studies on the applications of variable order fractional models for the flow stress behaviors of alloys. Therefore, the motivation of the present study is to propose a new method of variable order fractional model that can describe both strain softening and strain hardening phenomena of alloys. The fractional order in the functional form will be used, which can ensure the rationality of the numerical range and reveal the change in the properties of the deformation process. This study will provide a simple and unified constitutive model for the flow behaviors of alloys.

The rest of the present paper is arranged as follows. Section 2 will introduce the variable order fractional constitutive model for viscoelastic materials, and the stress-strain relationship for data fitting will be derived. Then in section 3, the model fitting method of the proposed model will be introduced. In section 4, the data fitting procedure will be carried out. The proposed model will be compared to the experimental data of the flow stress behavior of alloys, including both strain softening and strain hardening, and the rules of the parameters and order functions will be further analyzed. Finally, conclusions will be drawn in section 5.

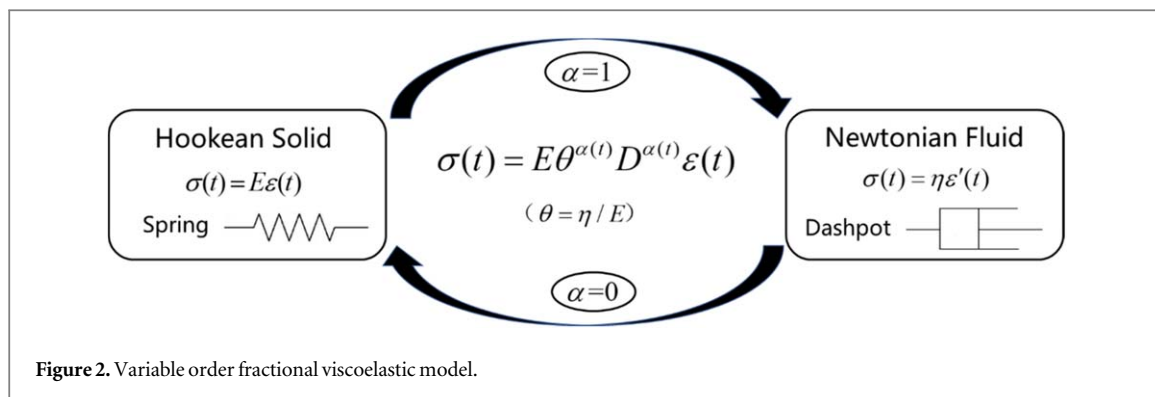


Figure 2. Variable order fractional viscoelastic model.

2. The variable order fractional constitutive model

The classical fractional order constitutive model for viscoelastic materials is proposed by Smit and Vries [23],

$$\sigma(t) = E\theta^\alpha D^\alpha \varepsilon(t) \quad 0 \leq \alpha \leq 1 \tag{1}$$

in which $E(\text{MPa})$ is the elastic modulus and $\theta(\text{s})$ is the relaxation time which can be written into the ratio of viscosity to elastic modulus $\theta = \eta/E$, D^α represents the fractional derivative operator of order α . It is not difficult to imagine that when $\alpha = 0$ equation (1) is equivalent to Hooke's law of ideal solid $\sigma(t) = E\varepsilon(t) = ED_t^0 \varepsilon(t)$, and when $\alpha = 1$ it will be Newton's law of viscosity $\sigma(t) = \eta d\varepsilon(t)/dt = \eta D_t^1 \varepsilon(t)$. Therefore, when the intermediate fractional order is used in equation (1), it can describe viscoelastic materials between solids and fluids. In the fractional order constitutive model, the fractional order operator has many different definitions. The most common ones are the Riemann-Liouville definition and the Caputo definition of fractional derivative, which are defined as

$${}^{RL}D_t^\alpha f(t) = \frac{1}{\Gamma(1-\alpha)} \frac{d}{dt} \int_\alpha^t \frac{f(\tau) d\tau}{(t-\tau)^\alpha} \tag{2}$$

$${}^CD_t^\alpha f(t) = \frac{1}{\Gamma(1-\alpha)} \int_0^t \frac{f'(\tau) d\tau}{(t-\tau)^\alpha} \tag{3}$$

where $\Gamma(*)$ is the Gamma function

$$\Gamma(t) = \int_0^t e^{-\tau} \tau^{t-1} d\tau \tag{4}$$

Compared with the Riemann-Liouville definition, the Caputo definition requires less involved treatments of the initial conditions and is more suitable for engineering applications.

Based on the fractional order constitutive model, considering the reality that the mechanical response of viscoelastic materials is continuously changing with the deformation process, Samko and Ross [24] further changed the order in the fractional viscoelastic model into the form of time function.

$$\sigma(t) = E\theta^{\alpha(t)} D^{\alpha(t)} \varepsilon(t) \tag{5}$$

Thus, the variable order fractional viscoelastic model can effectively describe the variation of mechanical properties between solids and fluids during deformation, as shown in figure 2.

The current definitions of variable order fractional operators are usually derived from the definitions of constant order fractional operators. Coimbra [17] proposed a variable order fractional operator by taking the Laplace-transform of Caputo's definition of fractional derivative.

$$D_0^{\alpha(t)} f(t) = \frac{1}{\Gamma[1-\alpha(t)]} \int_{0+}^t (t-\tau)^{-\alpha(t)} f^{(1)}(\tau) d\tau + \frac{(f(0+) - f(0-))t^{-\alpha(t)}}{\Gamma[1-\alpha(t)]} \tag{6}$$

If the fractional order is a constant, equation (6) is equivalent to the constant order Caputo definition, thus enabling a continuous and smooth transition between all integer order calculus operators [25]. Therefore, the operator in the variable order fractional viscoelastic model of this paper will use the Coimbra definition.

In studying the flow behavior of alloys, the uniaxial loading condition at the constant strain rate $\dot{\varepsilon}$ is usually used. Therefore, the strain function $\varepsilon(t) = \dot{\varepsilon}t$ is substituted into the variable order fractional operator of equation (6) to obtain

$$D^{\alpha(t)}f(t) = \frac{1}{\Gamma(1 - \alpha(t))} \int_0^t (t - \tau)^{-\alpha(t)} \dot{\varepsilon} d\tau \quad (7)$$

Integrating the right side of equation (7) yields

$$D^{\alpha(t)}f(t) = \frac{\dot{\varepsilon} t^{1-\alpha(t)}}{\Gamma(2 - \alpha(t))} \quad (8)$$

Substituting equation (8) into equation (5) gives

$$\sigma(t) = E(\theta)^{\alpha(t)} \frac{\dot{\varepsilon} t^{1-\alpha(t)}}{\Gamma(2 - \alpha(t))} \quad (9)$$

Finally, by substituting the constant strain rate $\varepsilon(t) = \dot{\varepsilon}t$ into equation (9), we can obtain a fitting equation in the form of stress-strain relationship as

$$\sigma(t) = E(\dot{\varepsilon}\theta)^{\alpha(t)} \frac{\varepsilon(t)^{1-\alpha(t)}}{\Gamma(2 - \alpha(t))} \quad (10)$$

3. Model fitting methods

In the variable order fractional viscoelastic model, the variable order function is crucial in describing the mechanical property change during deformation. Therefore, before applying the stress-strain relationship equation of equation (10) to describe the experimental data, the form of the variable order function must first be determined. In a previous work [26], the effects of four potential forms of order function, including piecewise, linear, trigonometric, and exponential, were studied in comparison. The results showed that linear function is more practical for mechanical modelling. Besides, the linear order function also directly reflects the way of constant strain rate loading for the flow behavior of alloys. As a result, the variable fractional order is assumed to vary linearly with time in this paper,

$$\alpha(t) = at + b \quad (11)$$

The variation of order with strain is also linear under constant strain rate loading conditions,

$$\alpha(\varepsilon) = a\varepsilon + b \quad (12)$$

Finally, by introducing equation (12) into equation (10), the stress-strain equation for data fitting can be written as

$$\sigma(\varepsilon) = E(\dot{\varepsilon}\theta)^{a\varepsilon+b} \frac{\varepsilon^{1-(a\varepsilon+b)}}{\Gamma(2 - (a\varepsilon + b))} \quad (13)$$

In equation (13), there are only four parameters, E , θ , a , and b , which can be easily obtained by data fitting. However, the mechanical response of alloys is significantly different before and after the yield point. Therefore, to reasonably describe the flow stress behaviors of alloys using the variable order fractional constitutive model, it is necessary to divide different stages by the yield point during the fitting procedure.

In this paper, the flow stress behaviors mainly concerned are strain softening and strain hardening. Taking strain softening as an example, there is a peak value in the stress-strain curve, after which the flow stress decreases. However, the peak value is not exactly equal to the yield point because alloys begin to yield before that. Generally speaking, the yield point of alloys is the intersection of the stress-strain curve and the parallel line of the elastic stage at the strain of 0.2%, as shown in figure 3. The stress-strain curve is then divided into the elastic stage and the strain softening stage, where the mechanical properties of alloys are significantly different due to the change in microstructure. As a result, when using the variable order fractional constitutive model, the stress-strain curve before and after the yield point should be fitted separately.

In order to fit the experimental data with the stress-strain relationship of equation (13), the toolbox 'Curve fitting tool' of MATLAB software is used, which employs the least square method to obtain the model parameters. Furthermore, the values of root mean squared error (RMSE) and coefficient of determination (R-square) will also be provided to evaluate the goodness of fit of the model.

In theory, through the above model fitting methods, the variable order fractional model is able to describe both the strain softening and strain hardening behaviors, and the variable order function can reflect the variation of mechanical property with temperature and strain rate. The validation of the proposed methods as well as the discussion of the model fitting results will be presented in the next section.

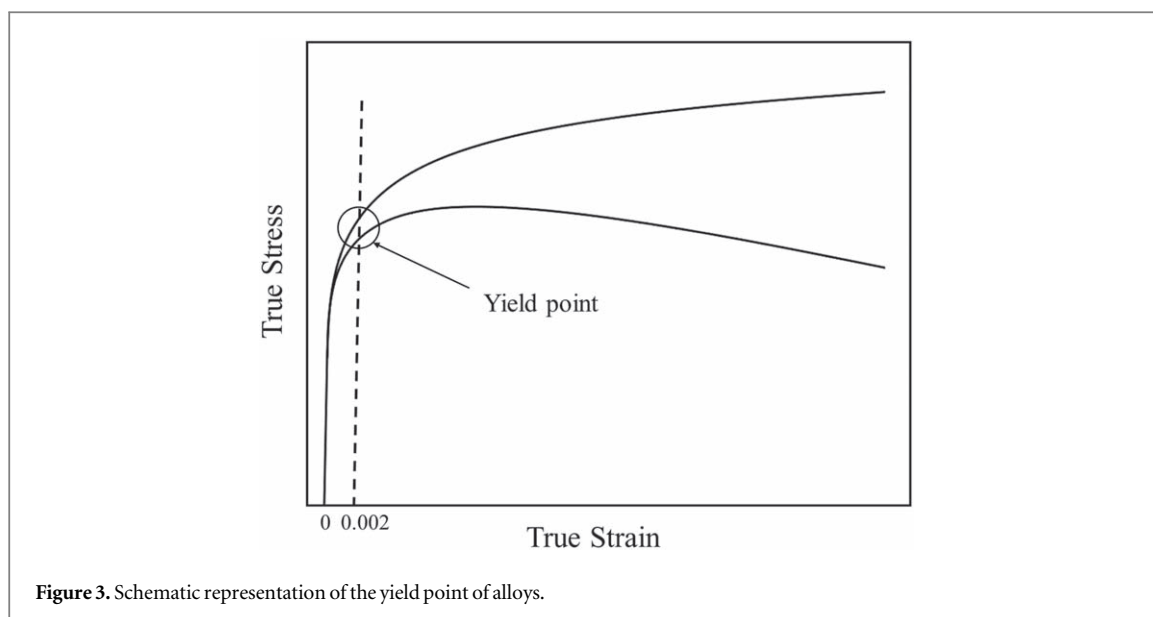


Figure 3. Schematic representation of the yield point of alloys.

Table 1. Model parameters and goodness of fit for A356 alloy at the temperature of 400 °C.

	Stage 1			Stage 2		
	$\dot{\varepsilon}$ (1 s) ⁻¹	0.1	0.01	0.001	0.1	0.01
E (MPa)	2990	1915	1469	29.53	11.51	6.06
θ (s)	0.137	1.78	23.06	30.67	474.4	6867
a	3.3	3.5	4.6	-6.3	-0.4	-0.38
b	0.69	0.75	0.8	0.88	0.94	0.98
RMSE	2.814	2.066	2.605	0.2232	0.2702	0.2515
R ²	0.9357	0.9405	0.9573	0.9961	0.992	0.9968

4. Validation and discussion

To validate the effect of the proposed model fitting methods for the variable order fractional model, the experimental data of alloys, including the strain softening and strain hardening behaviors, are used for data fitting.

4.1. Strain softening

Niu *et al* [27] researched the influence of deformation parameters on the strain softening of A356 alloy. They conducted the compression experiments at temperatures from 300 to 500 °C at varying strain rates. Here we selected the experimental data at the temperature of 400 °C with strain rates varying from 0.1 s⁻¹, 0.01 s⁻¹ to 0.001 s⁻¹, and the data at the strain rate of 0.01 s⁻¹ with temperatures ranging from 350 °C, 400 °C and 450 °C. Using the model fitting method in section 3, the stress-strain curves are fitted in two stages, with stage 1 before the yield point and stage 2 after that. The model parameters, as well as the goodness of fit of A356 alloy at the temperature of 400 °C, are presented in table 1. The comparison of model fitting results and the experimental data is shown in figure 4. It is clearly seen in figure 4 that the curves obtained by the proposed model are in good agreement with the experimental data of A356 alloy at the temperature of 400 °C, which has also been reflected by the values of RMSE and R-square given in table 1.

The same results are achieved for the experimental data of A356 alloy at the strain rate of 0.01 s⁻¹. The model parameters and goodness of fit are shown in table 2, and the data fitting results are plotted in figure 5. The model also fits well with the experimental data of A356 alloy at the strain rate of 0.01 s⁻¹ with temperatures from 350 °C, 400 °C to 450 °C. Generally speaking, the proposed variable order fractional model can accurately describe the strain softening behavior of A356 alloy at various temperatures and strain rates.

From table 1 and table 2, it can be obtained that the variation of material constants is in accordance with the influence of loading conditions on mechanical properties. As the temperature increases or the strain rate decreases, the elastic modulus E of A356 alloy decreases while the relaxation time θ increases, which means the

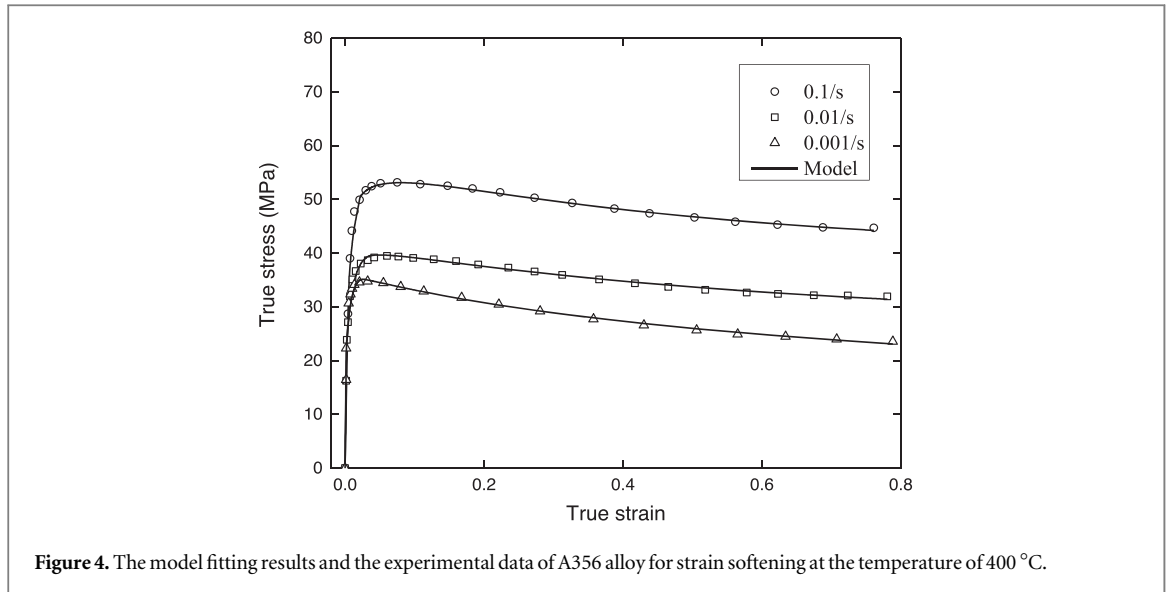


Figure 4. The model fitting results and the experimental data of A356 alloy for strain softening at the temperature of 400 °C.

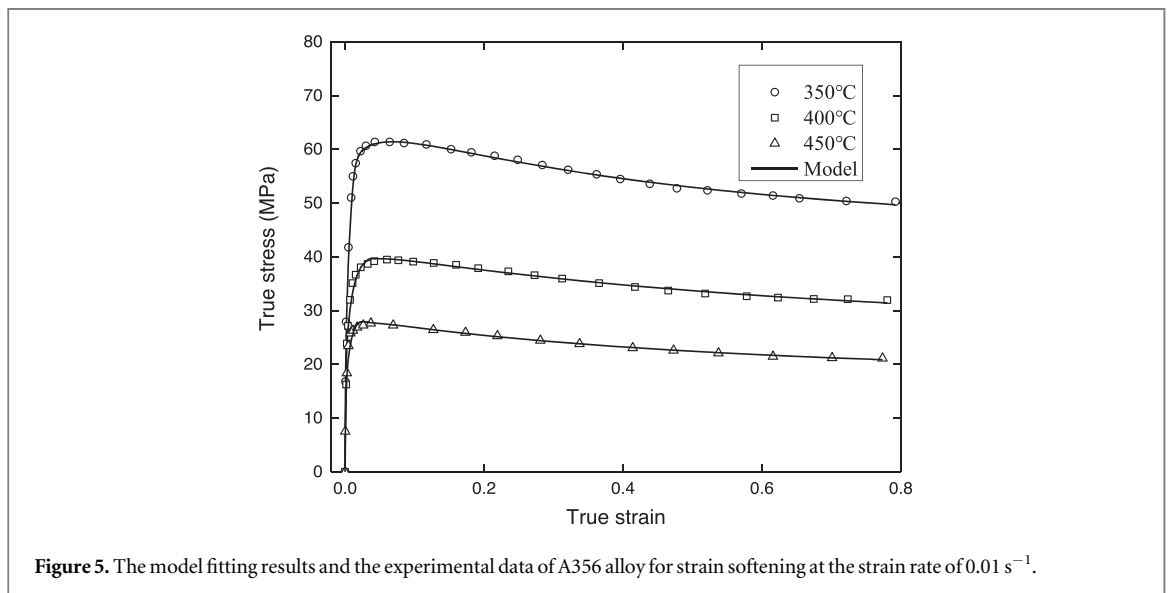


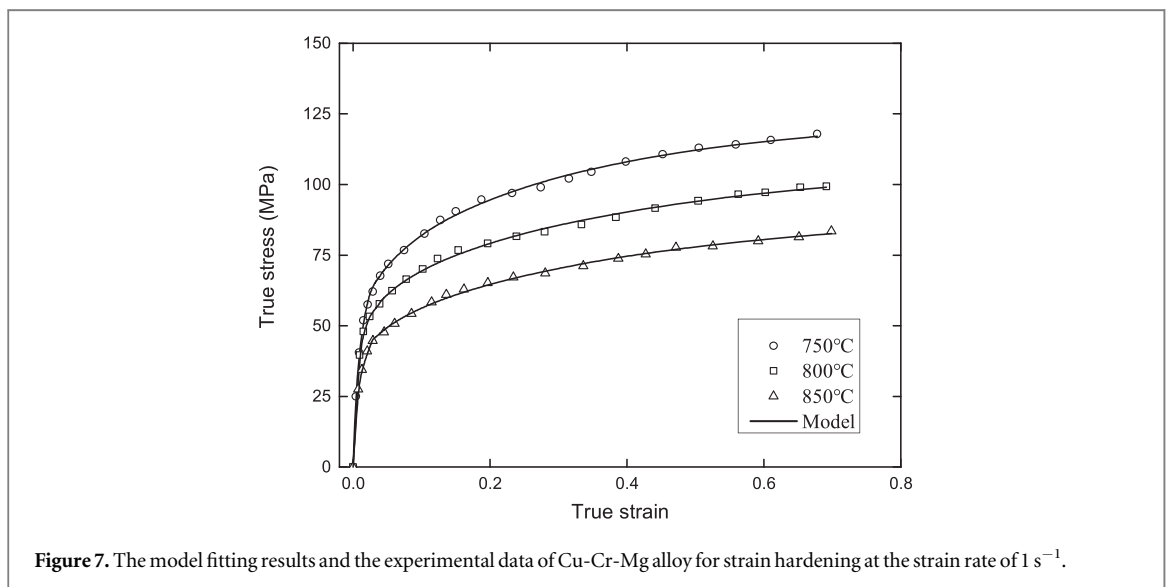
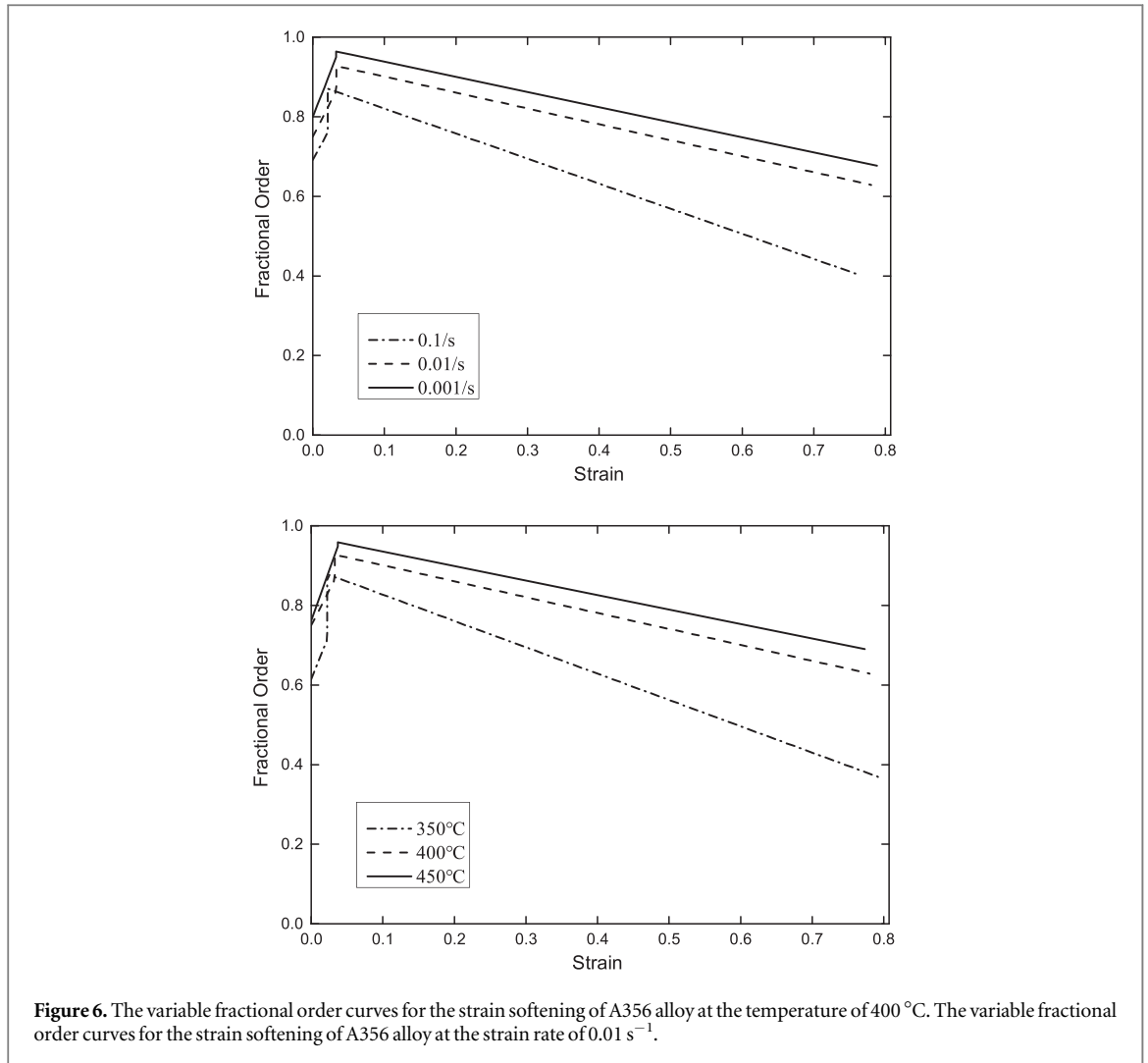
Figure 5. The model fitting results and the experimental data of A356 alloy for strain softening at the strain rate of 0.01 s^{-1} .

Table 2. Model parameters and goodness of fit for A356 alloy at the strain rate of 0.01 s^{-1} .

	Stage 1			Stage 2		
T (°C)	350	400	450	350	400	450
E (MPa)	7237	1915	1445	34.8	11.51	6.623
θ (s)	0.5	1.78	1.8	291.2	474.4	504.5
a	4.48	3.5	5	-0.66	-0.4	-0.36
b	0.61	0.75	0.76	0.89	0.94	0.97
RMSE	2.407	2.066	1.592	0.2953	0.2702	0.1591
R ²	0.9858	0.9405	0.9546	0.9953	0.992	0.996

strength of A356 alloy decreases under these conditions. Furthermore, as mentioned before, the fractional order function in the model can reveal the change of mechanical property, so the rule of the fractional order varying with strain should be further discussed.

The variable fractional order versus strain curves for the strain softening of A356 alloy at the temperature of 400 °C and the strain rate of 0.01 s^{-1} are plotted in figure 6 and figure 7, respectively. In terms of the overall values of the curves, the fractional order increases with increasing temperature and decreasing strain rate, indicating that the strength of A356 alloy decreases in these cases, which is consistent with the influence on the



material constants. The range of values is restricted between 0 and 1, which is in accordance with the definition of the fractional description of viscoelasticity. Moreover, it can be found in both figures that the fractional order increases before the yield point and continuously decreases in the strain softening stage. From the evolution of mechanical property, the smaller order means the material is closer to the fluid. This indicates that the elastic property of A356 alloy weakens until the yield point. Then with the strain softening stage developing, its

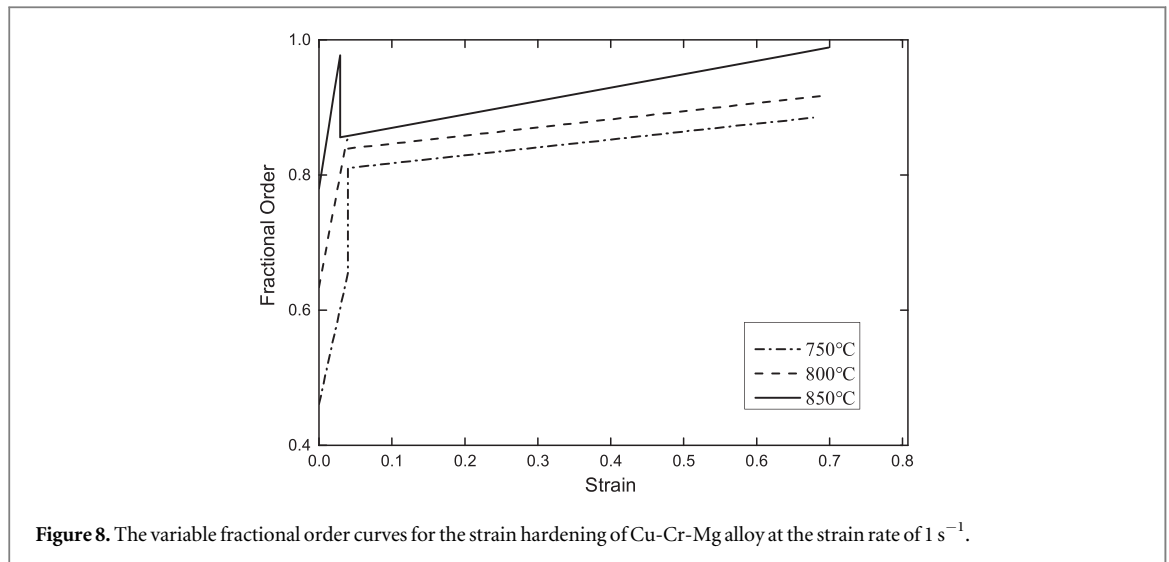


Figure 8. The variable fractional order curves for the strain hardening of Cu-Cr-Mg alloy at the strain rate of 1 s^{-1} .

Table 3. Model parameters and goodness of fit for Cu-Cr-Mg alloy at the strain rate of 1 s^{-1} .

	Stage 1			Stage 2		
T (°C)	750	800	850	750	800	850
E (MPa)	3705	1615	188.6	112.3	45	25.1
θ (s)	0.01	0.033	0.249	1.034	2.333	3.329
a	4.884	5.738	6.85	0.117	0.12	0.199
b	0.461	0.634	0.78	0.806	0.835	0.85
RMSE	1.837	1.283	0.7765	0.9706	0.8881	0.713
R^2	0.9971	0.9992	0.9986	0.9969	0.9958	0.9969

hardness gradually increases, and therefore the rate of stress reduction decreases. It can be concluded that the variable fractional order function vividly reveals the mechanical property evolution of A356 alloy during strain softening behavior.

4.2. Strain hardening

For alloys experiencing strain hardening behavior, Wang *et al* [28] performed isothermal compression tests on several kinds of alloys at various temperatures and strain rates. The experimental data of Cu-Cr-Mg alloy are chosen for data fitting, including three different temperatures of 750°C , 800°C , and 850°C at the strain rate of 1 s^{-1} . The model parameters and goodness of fit are listed in table 3. Similarly, the elastic modulus E is decreasing, and the relaxation time θ is increasing with the temperature increases, which means the strength of Cu-Cr-Mg alloy also reduces in this situation. The comparison between the model fitting results and the experimental data are presented in figure 7. It is still noticeable that the present model can give an accurate description of the strain hardening behavior of Cu-Cr-Mg alloy.

In addition, the variable fractional order versus strain curves are plotted in figure 8. The overall values of the fractional order functions follow the same rule as A356 alloy, which increases with increasing temperature within the range of 0 and 1. This suggests that the strength of Cu-Cr-Mg alloy is lower at higher temperatures, which also meets the rule of the material constants. However, the variation of the fractional order with strain is different from that of strain softening behavior. Although the fractional order curves are rising before the yield point, after yielding, they do not decrease as for strain softening behavior, but still rise linearly. This is due to the different trends in the evolution of mechanical properties of alloys during strain softening and strain hardening. During strain softening, the rate of stress reduction decreases, indicating that the hardness of alloys increases with strain, corresponding to the fractional order close to zero. While in strain hardening, the rate of stress increase decreases, indicating that the hardness decreases with strain and the corresponding fractional order increases toward 1. Therefore, we can say that the evolution of mechanical property of Cu-Cr-Mg alloy during strain hardening behavior has also been effectively described by the variable fractional order functions.

All in all, it is demonstrated that the variable order fractional model can accurately describe the flow stress behaviors of alloys by comparing the variable order fractional model with the experimental data of both strain

softening and strain hardening behaviors. The laws of the fractional order functions in the model can intuitively reveal the change of mechanical properties of alloys during the deformation process.

5. Conclusion

In the presented study, a variable order fractional constitutive model has been proposed for the flow stress behaviors of alloys during the hot working process. By assuming that the fractional order is linearly varying with time, the proposed model is able to describe both the strain softening and strain hardening behaviors. The effectiveness of the proposed method is validated by comparing the model fitting results with the experimental data of alloys under various temperatures and strain rates. The stress-strain curves of alloys are fitted separately before and after the yield point, which is the intersection of the stress-strain curve and the parallel line of the elastic stage at the strain of 0.2%. The conclusions obtained regarding the laws of the fractional order are as follows:

- (1) The overall values of the fractional order functions increase with increasing temperature or decreasing strain rate, which suggests that the strength of alloys is lower at higher temperatures or smaller strain rates.
- (2) Before the yield point, the fractional order is a linearly increasing function for both the strain softening and strain hardening behaviors. This means the elastic property of alloys weakens until the yield point.
- (3) After the yield point, the fractional order decreases for strain softening behavior, and increases for strain hardening behavior. This indicates that the hardness of alloys increases with strain during strain softening while decreasing with strain for strain hardening.

In summary, it is demonstrated that the variable order fractional model can accurately describe the flow stress behaviors of alloys, and the variation of the fractional order is able to reflect the evolution of mechanical property during the deformation process.

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Data availability statement

All data that support the findings of this study are included within the article (and any supplementary files).

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